Lower bounds on $\Delta M_{s,d}$ from constrained minimal flavour violation

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ABSTRACT: We demonstrate that in the class of models with Constrained Minimal Flavour Violation (CMFV), in which the CKM matrix is the only source of CP violation and $B_q^0 - \bar{B}_q^0$ mixings ($q = d, s$) are described by a single operator $(\bar{b}q)_{V-A}(\bar{b}q)_{V-A}$, the lower bounds on the mass differences $\Delta M_{s,d}$ are simply given by their Standard Model values $(\Delta M_{s,d})_{\rm SM}$ with two possible exceptions that we identify. Our proof involves all possible charged gauge boson, Goldstone boson and physical charged and neutral scalar exchanges and assumes that the masses of new charge $+2/3$ singlet heavy fermions T_i that mix with the top quark have all to be larger than m_t, which is chosen by nature. Similarly, the additional charged gauge bosons have to satisfy $M_i > M_W$, while no such bound needs to be set on the charged and neutral scalar masses. The two possible exceptions arise in the presence of U(1) neutral gauge bosons and Majorana fermions in box diagrams with flavour violating couplings. However in the case of the MSSM with CMFV and low $\tan\beta$ our bound is satisfied in spite of gluino and neutralino contributions.

KEYWORDS: B-Physics, Beyond Standard Model.

Contents

1. Introduction

The $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing mass differences $\Delta M_{d,s}$ played, in the last 20 years, a prominent role in the determination of the CKM matrix [1]. Simultaneously, being generated first at the one-loop level in weak interactions, $\Delta M_{d,s}$ played an important role in the tests of the short distance structure of the Standard Model (SM) and of its extensions, like the MSSM, models with Extra Dimensions, multi-Higgs models and Little Higgs models with and without T-parity.

With the advent of tree level methods for the determination of the CKM matrix [2], that, in contrast to $\Delta M_{d,s}$, are to a very good approximation independent of new physics contributions, the role of $\Delta M_{d,s}$ will be shifted primarily to the tests of the SM and its extensions.

One of the simplest classes of extensions of the SM are models with Constrained Minimal Flavour Violation (CMFV) [3] in which[1]

 (i) the SM Yukawa couplings and the resulting CKM matrix are the only source of flavour and CP violation,

 (ii) only operators are relevant that are relevant already in the SM.

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[1]A more general formulation of Minimal Flavour Violation can be found in [4]. Our present discussion applies only to CMFV as formulated in [3] and here.

Consequently, $\Delta M_{d,s}$ are governed by the CKM factors

$$\lambda_t^{(d)} = V_{tb}^* V_{td}, \qquad \lambda_t^{(s)} = V_{tb}^* V_{ts}, \tag{1.1}$$

and the single local operator in the effective theory is $(q = d, s)$

$$\mathcal{Q}_q = (\bar{b}\gamma_\mu(1 - \gamma_5)q) \otimes (\bar{b}\gamma^\mu(1 - \gamma_5)q). \tag{1.2}$$

Therefore, in this class of models one simply has [5]

$$\Delta M_q = \frac{G_F^2}{6\pi^2}\eta_B m_{B_q}\hat{B}_{B_q}F_{B_q}^2 M_W^2|S(y)||\lambda_t^{(q)}|^2, \tag{1.3}$$

where $\hat{B}_{B_q}F_{B_q}^2$ result from the non-perturbative evaluation of the relevant hadronic matrix elements of $\mathcal{Q}_q$, and η_B is the QCD correction that in the SM is found at the NLO level to be $\eta_B = 0.55$ [6].

The most important quantity in the present paper is the short distance function

$$S(y) = S_0(x_t) + \Delta S(y), \tag{1.4}$$

resulting from box diagrams. Here $S_0(x_t)$ with $x_t = m_t^2/M_W^2$, introduced in [7], is one of the so-called Inami-Lim functions [8] in the SM, and $\Delta S(y)$ summarizes all new physics contributions in a given CMFV model with y denoting collectively the parameters of this model. It should be emphasized that all CP-violating phases are collected in this class of models in $\lambda_t^{(q)}$ so that $S(y)$ is a real function. The explicit expression for $S_0(x_t)$ can be found in [5]. For the $\overline{\text{MS}}$ top quark mass $\overline{m}_t = (161.7 \pm 2.0)\,\text{GeV}$ one has $S_0(x_t) = 2.28 \pm 0.04$.

As already emphasized in [9], in all known CMFV models, $\Delta S(y)$ turns out to be positive so that $\Delta M_q > (\Delta M_q)_{\text{SM}}$ appears to be a property of this class of models. This is the case for Two-Higgs-Doublet Models type II [10], the MSSM with CMFV and low $\tan\beta$ [11, 12], the Littlest Higgs model (LH) without T-parity [13, 14], the CMFV limit of the LH model with T-parity [15] and Universal Extra Dimensions [16]. Concerning the MSSM with CMFV and low $\tan\beta$ analyzed in [11, 12], we would like to warn the reader, following [17], that these papers did not include gluino and neutralino contributions, and consequently the issue of negative contributions due to these particles being exchanged has not been addressed there. We will return to this issue in section 6.

On the other hand, in the MSSM with MFV and large $\tan\beta$, where new operators contribute to $\Delta M_{s,d}$, $\Delta M_s < (\Delta M_s)_{\text{SM}}$ in the full space of the parameters of the model [18]. In more complicated models, like the MSSM with new flavour violating interactions [19], or the LH model with T-parity [15], ΔM_s can be smaller or larger than $(\Delta M_s)_{\text{SM}}$.

On the experimental side both mass differences are known with high precision [20, 21]

$$\Delta M_d = (0.508 \pm 0.004)/\text{ps}, \qquad \Delta M_s = (17.77 \pm 0.10 \pm 0.07)/\text{ps} \tag{1.5}$$

with the latter measurement being one of the highlights of 2006. The SM is compatible with both values, even if the predictions for $(\Delta M_s)_{\text{SM}}$ prior to the measurements displayed the central values in the ballpark of 21/ps [22]. Unfortunately, due to sizable non-perturbative

uncertainties in $\hat{B}_{B_q} F_{B_q}^2$ [23], it will still take some time before we will know through the direct calculation of (1.3) whether $\Delta M_s > (\Delta M_s)_{\rm SM}$ or $\Delta M_s < (\Delta M_s)_{\rm SM}$ or even $\Delta M_s \simeq (\Delta M_s)_{\rm SM}$ with high precision. Some proposals for clarifying this issue with the help of the CP-violating asymmetry $S_{\psi\phi}$ and the semi-leptonic CP asymmetry $A_{\rm SL}^s$ have been put forward in [9], but they are very challenging as well.

Now, as stated above, in all CMFV models studied in the literature, $\Delta M_{d,s} > (\Delta M_{d,s})_{\rm SM}$ provided the CKM parameters have been determined by means of tree level decays, i.e. independently of new physics. The question then arises whether one could prove this to be a general property of the CMFV class by assuming

(i) three generations of the standard quarks (intrinsic property of CMFV anyway),

(ii) GIM mechanism [24], even in the presence of new heavy singlet T_i quarks with electric charge $+2/3$ that mix only with the standard top quark.

The main goal of the present paper is to demonstrate diagrammatically that indeed $(\Delta M_{d,s})_{\rm SM}$ is the lower bound for $\Delta M_{d,s}$ in the class of models with CMFV as defined in [3] that satisfy the two properties listed above, provided only charged gauge bosons, charged and neutral scalars and charged fermions are present in the box diagrams, and mild conditions on the spectrum of the gauge bosons and fermions are satisfied. The bound is also valid for tree-level neutral gauge boson exchanges, while tree-level scalar exchanges give different operators and are outside the CMFV framework. On the other hand we find that the presence of U(1) neutral gauge bosons and Majorana fermions[2] in the box diagrams with flavour violating couplings could in principle violate our bound. The neutral gauge boson contributions are strictly negative, but we are not aware of any model with neutral gauge boson contributions in box diagrams that could be put into the CMFV class. On the other hand, in the MSSM at low $\tan\beta$ with universal soft breaking terms introduced at a scale much higher than M_W, the presence of gluinos in box diagrams could in principle violate our bound. However a recent detailed analysis of the MFV limit of the MSSM at low $\tan\beta$ shows that this does not happen [17]. Moreover, as pointed out in that paper, the MSSM with MFV and low $\tan\beta$ does not belong to the CMFV class, if the higgsino coupling is large compared to the other SUSY scales. We will elaborate on this in section 6.

Our paper is organized as follows. In section 2 we will consider contributions from charged gauge bosons, Goldstone bosons and physical charged scalars. In section 3 we discuss how the measured Fermi constant can be modified by new physics contributions and study the implications on the predicted values for $\Delta M_{d,s}$. In section 4 we study the contributions of an arbitrary number of heavy singlet T_i quarks that mix with the top quark. In section 5 we show that the above cases cover indeed all charged particle contributions possible within the CMFV framework. In section 6 we address the issue of neutral particle exchanges (gauge bosons, fermions and scalars) advertised above. We briefly summarize our work in section 7.

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[2]We thank Diego Guadagnoli and Gino Isidori for bringing this case to our attention.
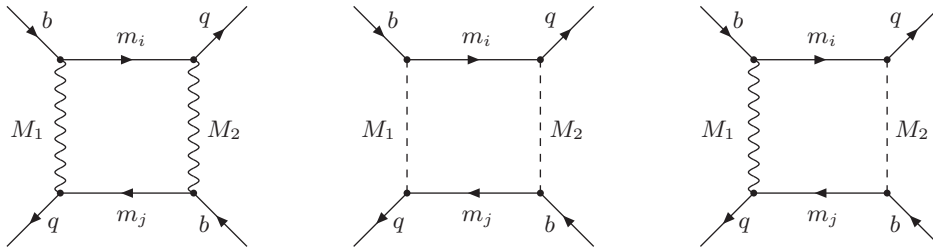
Figure 1: Classes of diagrams contributing to $\Delta B = 2$ processes in CMFV models.

2. Contributions from charged gauge bosons and scalars

For our purposes it will be sufficient to consider the three box diagrams shown in figure 1 and calculate them in the 't Hooft-Feynman gauge. The first diagram, D_1, describes the contribution of two gauge bosons with masses M_1 and M_2 and ordinary up-type quarks. The second diagram, D_2, corresponds to two scalar exchanges that can be physical scalars and Goldstone bosons. Finally, the third diagram, D_3, gives mixed gauge and scalar contributions.

In giving the formulae below we will simply multiply the result of each diagram by a factor of i to obtain its contribution to the effective Hamiltonian for $\Delta B = 2$ transitions, which takes the general form

$$\mathcal{H}_{\text{eff}}(\Delta B = 2) = c \left(\lambda_t^{(q)} \right)^2 S(y) \mathcal{Q}_q \,, \tag{2.1}$$

with c being a positive constant that includes for instance G_F and QCD corrections. We recall that

$$\Delta M_q = \frac{\left| \langle \bar{B}_q^0 | \mathcal{H}_{\text{eff}}(\Delta B = 2) | B_q^0 \rangle \right|}{m_{B_q}} \,. \tag{2.2}$$

In what follows we will not include combinatorial factors like a factor of 2 for two different bosons being exchanged, nor QCD corrections, as they do not have any impact on our basic result. Indeed, as we will demonstrate below, each of the diagrams in figure 1 gives a positive contribution to $\mathcal{H}_{\text{eff}}$ in (2.1) times the relevant CKM factor. Taking next into account the contributions from ordinary u, c, t quarks, with $m_u, m_c \ll m_t$, relevant for the $\lambda_t^{(q)}$ term in (1.3), and using the unitarity of the CKM matrix, we show that the known GIM structure in the case of $\Delta B = 2$ processes

$$S(y) \propto F(t,t) + F(u,u) - 2F(u,t) \,, \tag{2.3}$$

resulting from each diagram in figure 1 independently, remains positive, which completes the proof. Note that by extending the theory with additional gauge bosons and scalars, the SM part remains unchanged and the new physics leads simply to additional positive contributions. Here $F(i,j)$ denotes the short distance function up to a common overall factor that corresponds to a box diagram with i and j quark exchanges.

Calculating then the diagrams D_1, D_2 and D_3 of figure 1 in the 't Hooft-Feynman gauge, and multiplying them by i, we find $(i, j = u, c, t)$

$$iD_1 = i\frac{g_1^2 g_2^2}{16}\lambda_i^{(q)}\lambda_j^{(q)}I_1(m_i, m_j; M_1, M_2)\mathcal{Q}_q, \tag{2.4}$$

$$iD_2 = i\frac{k_{i1}k_{i2}k_{j1}k_{j2}}{4}\frac{g_1^2 g_2^2}{16}\lambda_i^{(q)}\lambda_j^{(q)}I_1(m_i, m_j; M_1, M_2)\mathcal{Q}_q, \tag{2.5}$$

$$iD_3 = -ik_{i2}k_{j2}m_i m_j\frac{g_1^2 g_2^2}{16}\lambda_i^{(q)}\lambda_j^{(q)}I_2(m_i, m_j; M_1, M_2)\mathcal{Q}_q. \tag{2.6}$$

Here, g_i are the gauge couplings, with $g_1 = g_2 = g$ in the SM, that in some extensions of the SM should be replaced in D_2 and D_3 by other model dependent couplings, which however, due to the CMFV hypothesis, always have to be flavour universal. The factors k_{ia} ($i = u, c, t$; $a = 1, 2$) denote the scalar-fermion couplings. In the SM, they are simply given by $k_{ia} \propto m_i$. In a general CMFV model they can be written as

$$k_{ia} = \alpha_a + \beta_a m_i + \mathcal{O}(m_i^2), \tag{2.7}$$

where $\alpha_a, \beta_a \in \mathbb{R}$ are flavour universal constants that may depend on the scalar a.

Finally, the two integrals I_1 and I_2 are given as follows

$$I_1(m_i, m_j; M_1, M_2) = \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_1^2)(k^2 - M_2^2)}, \tag{2.8}$$

$$I_2(m_i, m_j; M_1, M_2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_i^2)(k^2 - m_j^2)(k^2 - M_1^2)(k^2 - M_2^2)}. \tag{2.9}$$

Using the standard techniques, i.e. Wick rotation and integration over the angular directions, they can be rewritten as follows

$$iI_1(m_i, m_j; M_1, M_2) = \frac{1}{16\pi^2}\int_0^\infty dr \frac{r^2}{(r + m_i^2)(r + m_j^2)(r + M_1^2)(r + M_2^2)}, \tag{2.10}$$

$$-iI_2(m_i, m_j; M_1, M_2) = \frac{1}{16\pi^2}\int_0^\infty dr \frac{r}{(r + m_i^2)(r + m_j^2)(r + M_1^2)(r + M_2^2)}, \tag{2.11}$$

implying that all three expressions in (2.4)-(2.6) are, up to CKM factors $\lambda_{i,j}^{(q)}$, real and positive quantities.

This result remains true for the linear combination in (2.3) that enters the function $S(y)$ after the CKM unitarity relation

$$\lambda_u^{(q)} + \lambda_c^{(q)} + \lambda_t^{(q)} = 0 \tag{2.12}$$

has been used.

In order to see this in the case of pure gauge boson contributions, it is sufficient to look at the two quark propagators in (2.10). Then the linear combination in (2.3) reads

$$\frac{1}{(r + m_t^2)^2} + \frac{1}{r^2} - \frac{2}{r(r + m_t^2)} = \left(\frac{1}{r} - \frac{1}{r + m_t^2}\right)^2 > 0. \tag{2.13}$$

In the case of D_2, we find

$$\frac{(k_{t1}k_{t2})^2}{(r+m_t^2)^2} + \frac{(k_{u1}k_{u2})^2}{r^2} - \frac{2k_{u1}k_{u2}k_{t1}k_{t2}}{r(r+m_t^2)} = \left(\frac{k_{u1}k_{u2}}{r} - \frac{k_{t1}k_{t2}}{r+m_t^2}\right)^2 > 0\,. \qquad (2.14)$$

In the case of D_3 the functions $F(u,u)$ and $F(u,t)$ vanish due to $m_u = 0$, and as $F(t,t) > 0$, we again obtain positive contributions.

We have thus shown that, after the GIM mechanism has been used, all three classes of contributions, represented by the three diagrams in figure 1, give separately positive contributions to the function $S(y)$. This means that all new physics contributions of this class have the same sign as the SM contribution, which implies $\Delta S(y) > 0$. We have verified that the same result follows in the unitary gauge, where only physical scalars enter the diagrams in figure 1.

The above calculation applies not only to the three ordinary quark generations, but also to their heavy partners, that do not mix with the standard ones. This happens e.g. in models with Universal Extra Dimensions [16], where the heavy partners of the SM particles are the Kaluza-Klein modes of the latter. In this context it is important to note that the Yukawa couplings of the heavy partner fields have to be proportional to the masses of the corresponding SM quarks, in order not to violate CMFV. Note that the masses $m_i m_j$ multiplying D_3 now have to be replaced by the heavy partner masses. A straightforward calculation following the lines of (2.13) and (2.14) shows that also in this case the contribution from D_3 is positive.

We would like to remark that this proof also applies to chargino contributions in supersymmetric models with CMFV and low $\tan\beta$, in which the supersymmetric partners of the SM fields contribute only through diagrams of the type D_2. Note that in this case the GIM mechanism runs over the squarks, i.e. the scalars present in the box diagram D_2. Moreover, the index a in k_{ia} of (2.7) now corresponds to the charginos. The property of $\Delta S(y) > 0$ in the MSSM with CMFV and low $\tan\beta$ has been pointed out in [11] and confirmed in [12], but as already stated above these papers considered only Dirac fermion contributions, as we did in this section. The case of Majorana fermions is discussed in section 6.

3. Modification of G_F

Until now we have assumed that the SM gauge coupling g is known from experiment. This is however not exactly true. What is measured is the $W^\pm$ boson mass M_W and the Fermi constant G_F, obtained from the decay $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ and given in the SM by

$$G_F = \frac{g^2}{4\sqrt{2}M_W^2}\,. \qquad (3.1)$$

However, new charged gauge bosons and charged scalars also contribute to this decay, so that the expression for the measured G_F^{eff} in terms of g^2 gets modified as follows (see [13]

for a discussion of this issue in the LH model)

$$G_F^{\text{eff}} = \frac{g^2}{4\sqrt{2}M_W^2} + \sum_i \frac{g_i^2}{4\sqrt{2}M_i^2} - \sum_j \frac{g_j^{\Phi\,2}}{4\sqrt{2}M_j^{\Phi\,2}}\,, \tag{3.2}$$

implying a modified value of the determined g^2.

Here we have assumed the presence of additional gauge bosons $W_i^\pm$ with masses M_i and gauge couplings g_i and additional physical scalars $\Phi_j^\pm$ with couplings g_j^Φ and masses M_j^Φ. We find that additional gauge bosons lead to positive contributions to G_F^{eff}, while physical scalars tend to lower its value. This is due to the relative minus sign of the scalar propagator with respect to the gauge boson propagator. Consequently the presence of scalar contributions to G_F^{eff} always implies an increased value of g^2 and thus a further enhancement of $\Delta M_{d,s}$.

So, in the following we will concentrate only on additional gauge bosons present in the theory. In this case, solving (3.2) for g^2, we find

$$g^2 = 4\sqrt{2}M_W^2 G_F^{\text{eff}} - \sum_i \frac{M_W^2}{M_i^2} g_i^2\,. \tag{3.3}$$

Inserting this into the contributions of $W^\pm$ and $W_i^\pm$ to the integrand in (2.10), (2.11) leads to

$$\left(\frac{g^2}{r+M_W^2} + \sum_i \frac{g_i^2}{r+M_i^2}\right)^2 = \left(\frac{4\sqrt{2}G_F^{\text{eff}}M_W^2}{r+M_W^2} - \sum_i \frac{M_W^2}{M_i^2}\frac{g_i^2}{r+M_W^2} + \sum_i \frac{g_i^2}{r+M_i^2}\right)^2$$

$$= \left(\frac{4\sqrt{2}G_F^{\text{eff}}M_W^2}{r+M_W^2} + \sum_i \frac{g_i^2}{M_i^2}\frac{r(M_i^2-M_W^2)}{(r+M_W^2)(r+M_i^2)}\right)^2\,. \tag{3.4}$$

The first term is exactly the SM expression, and the second term is clearly positive provided $M_i > M_W$.

4. New heavy quarks T_i

We will now assume the existence of an arbitrary number of heavy singlet quarks T_i with electric charge $+2/3$ that can mix with the top quark. For simplicity we will suppress the mixing between the T_i's, but from the arguments given below it will be evident that our proof is valid also for this more complicated case.

It is well known [25] that the mixing of new singlet heavy quarks with the standard quarks implies the presence of FCNC transitions at tree level. Consequently this case, strictly speaking, does not belong to the CMFV framework. However, as far as K and B physics is concerned, the mixing of singlet T_i quarks with the top quark does not imply FCNC contributions at tree level, and thus models of this type belong to the CMFV class if only processes with external down-type quarks are considered.

The gauge-fermion vertices take now the general form ($q = d, s, b$)

$$\bar{t}qW^\mu \quad : \quad i\frac{g}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)V_{tq}c_t \tag{4.1}$$

$$\bar{T}_iqW^\mu \quad : \quad i\frac{g}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)V_{tq}s_i \tag{4.2}$$

with $0 < c_t < 1$ signalling the violation of the 3×3 CKM unitarity. For instance in the LH model we have at $\mathcal{O}(v^2/f^2)$ [13]

$$c_t = 1 - \frac{x_L^2}{2}\frac{v^2}{f^2}, \qquad s_T = x_L\frac{v}{f}, \tag{4.3}$$

where $v = 246\,\text{GeV}$ is the SM Higgs VEV, $f \sim 1\,\text{TeV}$ the new physics scale and $0 < x_L < 1$ a parameter of the LH model. Generalizing the discussion in [13] to an arbitrary number of T_i fields, the unitarity relation involving also T_i is given by

$$\lambda_u^{(q)} + \lambda_c^{(q)} + c_t^2\lambda_t^{(q)} + \sum_i s_i^2\lambda_t^{(q)} = 0 \tag{4.4}$$

with the "bare" $\lambda_i^{(q)}$ still satisfying (2.12). Therefore we also have

$$c_t^2 + \sum_i s_i^2 = 1. \tag{4.5}$$

We will now generalize (2.3) to include the mixing with the heavy quarks T_i.

Including u, c, t, T_i quark contributions to the box diagrams we find the following linear combination (we suppress the index q of $\lambda_i^{(q)}$) for each diagram in figure 1 that now includes also T_i exchanges:

$$\lambda_t^2 S(y) \propto c_t^4\lambda_t^2 F(t,t) + \lambda_c^2 F(c,c) + \lambda_u^2 F(u,u) + \sum_i s_i^4\lambda_t^2 F(T_i,T_i)$$

$$+ 2c_t^2\lambda_c\lambda_t F(c,t) + 2c_t^2\lambda_u\lambda_t F(u,t) + 2\lambda_u\lambda_c F(u,c) + \sum_{i\neq j} s_i^2 s_j^2\lambda_t^2 F(T_i,T_j)$$

$$+ 2\sum_i s_i^2 c_t^2\lambda_t^2 F(T_i,t) + 2\sum_i s_i^2\lambda_u\lambda_t F(T_i,u) + 2\sum_i s_i^2\lambda_c\lambda_t F(T_i,c). \tag{4.6}$$

Using $m_u, m_c \ll m_t$ and $\lambda_u + \lambda_c = -\lambda_t$ gives then

$$S(y) \propto c_t^4 F(t,t) + F(u,u) - 2c_t^2 F(u,t) + \sum_i s_i^4 F(T_i,T_i)$$

$$+ \sum_{i\neq j} s_i^2 s_j^2 F(T_i,T_j) + 2\sum_i s_i^2 c_t^2 F(T_i,t) - 2\sum_i s_i^2 F(T_i,u). \tag{4.7}$$

This linear combination is the generalization of the GIM structure (2.3) to include the mixing of t with an arbitrary number of heavy quarks T_i. In the case of a single T quark, (4.7) reduces to the linear combination (4.14) in [13].

Clearly the basic structure of diagrams contributing here is the same as in section 2. Consequently it is sufficient to look only at fermion propagators as in (2.13). With (4.7) we find now instead of (2.13)

$$\frac{c_t^4}{(r+m_t^2)^2} + \frac{1}{r^2} - \frac{2c_t^2}{r(r+m_t^2)} + \sum_i \frac{s_i^4}{(r+m_{T_i}^2)^2} + \sum_{i\neq j} \frac{s_i^2 s_j^2}{(r+m_{T_i}^2)(r+m_{T_j}^2)}$$

$$+ \sum_i \frac{2s_i^2 c_t^2}{(r+m_{T_i}^2)(r+m_t^2)} - \sum_i \frac{2s_i^2}{r(r+m_{T_i}^2)}$$

$$= \left(\frac{1}{r} - \frac{c_t^2}{r+m_t^2} - \sum_i \frac{s_i^2}{r+m_{T_i}^2} \right)^2$$

$$= \left(\frac{1}{r} - \frac{1}{r+m_t^2} + \sum_i s_i^2 \left[\frac{1}{r+m_t^2} - \frac{1}{r+m_{T_i}^2} \right] \right)^2, \quad (4.8)$$

where in the last step we have used the relation (4.5). The first two terms represent the SM contribution. The new physics contribution represented by the remaining terms in (4.8) is clearly positive provided $m_{T_i} > m_t$.

In the case of scalar exchanges, represented by D_2 and D_3, the u-quark contributions vanish due to $m_u \simeq 0$ and we find from (4.7)

$$\left(\frac{c_t^2 m_t^2}{r+m_t^2} + \sum_i \frac{s_i^2 m_{T_i}^2}{r+m_{T_i}^2} \right)^2 = \left(\frac{m_t^2}{r+m_t^2} + \sum_i s_i^2 \frac{r(m_{T_i}^2 - m_t^2)}{(r+m_{T_i}^2)(r+m_t^2)} \right)^2 \quad (4.9)$$

with the first term representing the SM contribution. Thus again provided $m_{T_i} > m_t$ the new physics contribution is positive. We would like to remark that here it is sufficient to consider the SM case $k_{ia} \propto m_i$, as the presence of additional scalars with more general couplings $\alpha_a + \beta_a m_i$ simply results in an additional positive contribution, which can straightforwardly be shown following the lines of section 2.

In summary, we have shown that $\Delta S(y) > 0$ also in the presence of a mixing of t with an arbitrary number of T_i fermions, provided $m_{T_i} > m_t$, as certainly chosen by nature.

5. Are there any additional contributions in CMFV?

Although we have so far considered contributions from new charged gauge bosons and charged scalars and from additional singlet T_i quarks separately, it is obvious that our proof also holds if both types of new particles are present simultaneously.

Let us now have a look whether we have missed any type of new physics, involving contributions from charged particles, consistent with the CMFV hypothesis as defined in [3] and satisfying the two properties listed in the introduction. In the next section we will investigate what happens if also neutral particles are considered.

- First, one could think of introducing additional sequential heavy quark doublets into the theory. This would, however, induce $N \times N$ flavour mixing with $N > 3$ and consequently new sources of flavour violation beyond the CKM matrix.

- Second, let us consider additional heavy singlet quarks B_i with charge $-1/3$, as present e.g. in grand unified theories based on the group E_6. These quarks cannot contribute to the box diagrams in figure 1, but their mixing with d, s, b generates a small violation of the CKM unitarity and consequently leads to tree level FCNC contributions to $\Delta B = 2$ processes. Clearly such processes are outside the CMFV framework. A detailed discussion of this issue can be found in [25].

6. Neutral particle contributions

Until now we have considered only flavour violating charged particle exchanges. As we have shown in this case our bound is clearly satisfied. In the present section we will have a close look at neutral particle contributions. Such contributions are present in the MSSM, where gluinos and neutralinos can be present in box diagrams. Such exchanges are also present in models with new sources of flavour violation and it is of interest to investigate whether some general statements on the sign of their contributions to $\Delta M_{s,d}$ can be made by assuming as in the CMFV models that only the operator $\mathcal{Q}_q$ in (1.2) is relevant. If the relevant flavour violating couplings were equal to the CKM couplings, a contribution like this could mimic contributions present in CMFV models, even if the flavour violation did not have anything to do with the SM Yukawa couplings. An example is provided by the Littlest Higgs model with T-parity (LHT) [15] if we choose the mixing matrix V_{Hd}, that parametrizes the interactions of down-quarks and mirror quarks, to be equal to the CKM matrix. In doing this one has to make sure that $\lambda_c^{(q)^2}$ contributions to $\Delta M_{s,d}$ are made to be negligible by taking the mirror fermions of the first two generations to be quasi-degenerate in mass. Otherwise the general structure in (1.3) would be modified. Our discussion below of neutral particle contributions is more general than the LHT model.

First, $\Delta B = 2$ processes could be mediated directly by tree level FCNCs, as in flavour violating Z' models [26]. This corresponds, however, to a new source of flavour violation that has nothing to to with the CKM matrix and CMFV. Even if accidentally the flavour changing couplings of Z' were equal to $\lambda_t^{(q)}$, a straightforward tree diagram calculation shows that also in this case $\Delta S(y) > 0$.

If, on the other hand, the neutral gauge bosons are exchanged in box diagrams, in addition to the usual diagram on the left in figure 2 also "crossed" box diagrams on the right in figure 2 contribute.

Let us first consider the case of a neutral gauge boson corresponding to a U(1) group.[3] First, as already demonstrated in [15, 27] in the context of the LHT model, the Goldstone boson contributions cancel between the two diagrams in figure 2 so that it suffices to consider only gauge boson contributions in the 't Hooft-Feynman gauge. Then the Dirac structure of the vertices in the box diagrams assures that the mass terms in the numerators of the fermion propagators do not contribute. The first diagram gives then a positive contribution to the function $S(y)$ as in the case of charged gauge boson contributions,

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[3]We assume that the possible anomalies related to this gauge group in the effective theory are cancelled by new fields in the full theory.
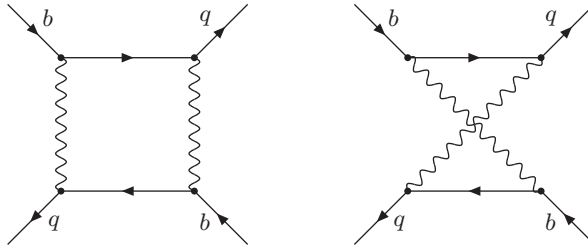
Figure 2: Neutral gauge boson contributions to $\Delta B = 2$ processes.

while the second one, having a negative sign in the four-momentum in the right fermion propagator, gives a *negative* contribution to $S(y)$. As in the first diagram

$$\gamma_\mu \gamma_\rho \gamma_\nu \otimes \gamma^\nu \gamma^\rho \gamma^\mu = 4\gamma_\mu \otimes \gamma^\mu \,, \tag{6.1}$$

but

$$\gamma_\mu \gamma_\rho \gamma_\nu \otimes \gamma^\mu \gamma^\rho \gamma^\nu = 16\gamma_\mu \otimes \gamma^\mu \tag{6.2}$$

in the second diagram, the latter diagram wins this competition and we find a strictly negative contribution to the function $S(y)$ that carries an overall factor $1 - 4 = -3$, which will play some role in a moment.

Thus we found a simple mechanism for a negative contribution to $S(y)$, which would be welcome if one could convincingly demonstrate that $\Delta M_s < (\Delta M_s)_{\mathrm{SM}}$.

Let us now consider the case of a neutral gauge bosons belonging to an adjoint representation of a simple non-abelian group like $\mathrm{SU}(N)$ that does not mix with neutral gauge bosons corresponding to additional $\mathrm{U}(1)$ gauge group factors. As we consider here gauge bosons which can contribute to $S(y)$ only at the loop level, the fermions on the internal lines of the box diagrams have to be different from the SM quarks. Such "mixed" gauge couplings can only result from the mixing of a new gauge group with the SM $\mathrm{SU}(2)_L$ one. This new gauge group thus has to be another $\mathrm{SU}(2)$, or an $\mathrm{SU}(2)$ subgroup of a larger group. In any way, the only new heavy gauge group which can appear in diagrams with new heavy fermions being exchanged is $\mathrm{SU}(2)$. Consequently, we will restrict our attention to this case in the following.

Now, also the positive charged gauge boson contributions have to be taken into account. Moreover, there is one charged, to be called $W_H^\pm$, and one neutral gauge boson, Z_H^0, present in $\mathrm{SU}(2)$ (note that $W_H^\pm$ has to be counted only once) and the vertices involving the Z_H^0 gauge boson carry an additional suppression factor $1/\sqrt{2}$ relative to the vertices involving the charged gauge boson $W_H^\pm$. Thus, the total neutral gauge boson contribution from the diagrams in figure 2 carries an overall factor $-3/4$ relative to $+1$ of the charged gauge boson contribution, when the 't Hooft-Feynman gauge is considered. As the charged Goldstone boson contributions have been shown in section 2 to be positive and the neutral ones do not contribute, the total contribution of gauge bosons of a simple non-abelian gauge group including Goldstone bosons to $S(y)$ is strictly positive.
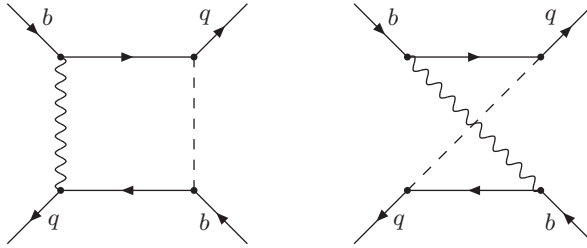
Figure 3: Neutral scalar contributions to $\Delta B = 2$ processes.

If the gauge group involves both SU(2) and U(1) factors and there is a mixing between neutral gauge bosons belonging to different group factors in the process of spontaneous symmetry breakdown, the situation is clearly more involved and a total negative contribution cannot be excluded if the number of U(1) factors is sufficiently high and the relevant U(1) charges large. Yet, the neutral gauge bosons have to compete not only with charged gauge boson contributions but also with related charged Goldstone boson contributions, that generally are more important than gauge boson contributions if there are heavy fermions in box diagrams. Thus also in this more general case it is very likely that the net contribution to the function $S(y)$ is positive.

An explicit example of this more complicated situation is provided by the LHT model. For the choice $V_{Hd} = V_{\text{CKM}}$ one finds that the $W_H^\pm$ contributions win over the joint Z_H^0 and A_H^0 contributions so that in this case $\Delta S(y) > 0$, but we cannot exclude that in the presence of several U(1) factors $\Delta S(y) < 0$ could be possible.

Let us now have a look at physical neutral scalars contributing to $\Delta B = 2$ processes. While the "direct" and "crossed" diagrams involving two scalar exchanges again cancel each other as in the case of neutral Goldstone bosons, this is generally not the case for the mixed gauge boson — scalar contributions as shown in figure 3. The reason is that in contrast to the neutral Goldstone bosons, whose couplings to fermions are always real, the vertices of physical neutral scalars to fermions can in principle be both real and imaginary. In the case of real couplings, the two diagrams in figure 3 again cancel each other, because the lower scalar fermion vertices in these two diagrams differ by sign, and we find that such scalars do not contribute to $S(y)$. On the other hand, in the case of imaginary couplings, these vertices are obviously equal, and we thus find that in this case the two diagrams simply add up and bring in a factor of 2. As the first diagram in figure 3 has been shown in section 2 to give a positive contribution to $S(y)$, we finally conclude that physical neutral scalars can only enhance $S(y)$, depending on their couplings to fermionic fields.

Finally, the case of neutral fermion contributions should be considered. In the case of Dirac fermions, which have a specified direction of fermion flow, only the first diagram in figure 4 contributes and gives a positive contribution to $S(y)$. In the case of Majorana fermions however, again direct and crossed diagrams are present, as shown in figure 4, with positive and negative contributions to $S(y)$, respectively. The counting is different from the one considered above as now scalars and Majorana fermions instead of Dirac fermions
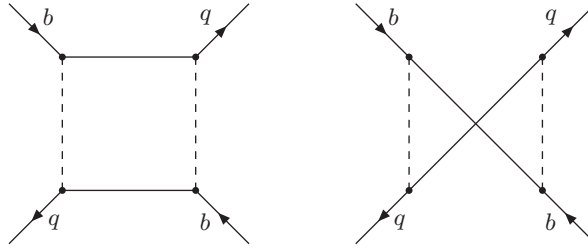
Figure 4: Majorana fermion contributions to $\Delta B = 2$ processes.

and gauge bosons are exchanged. Moreover, the nature of Majorana fermions complicates the issue and we were not able to reach a general clear cut conclusion as in the case of gauge bosons and scalars. On the other hand, a recent detailed analysis of the MFV limit of the MSSM at low $\tan\beta$ [17] shows that at least in this case if the higgsino coupling is large compared to the other SUSY scales the presence of Majorana fermions (gluinos and neutralinos) implies the presence of new operators so that this model, in contrast to the frequently made statements in the literature, does not belong to the CMFV class, and therefore it in principle does not concern our present analysis. However, a remarkable finding of [17] is that within the MFV MSSM at low $\tan\beta$ our bound holds even when new operators give non-negligible contributions. This feature is due to an interesting interplay between chargino and gluino box diagrams in the different mass regimes considered in [17]. Charginos always give positive contributions, whereas gluino contributions can be negative or positive. However, when the latter are negative, they are always outpaced by large positive chargino contributions, while when the chargino contributions are small, the gluino contributions turn out to be positive. Finally, neutralino contributions to $\Delta M_{s,d}$ are always negligible.

7. Summary

In this paper we have demonstrated that in any model with CMFV whose low energy limit is given by the SM, the new physics contributions to $\Delta M_{s,d}$ from charged particles are positive, so that $(\Delta M_{s,d})_{\text{SM}}$ represents the lower bound for $\Delta M_{s,d}$ in this case.

The validity of the bound in question is independent of the scalar masses. As far as the gauge sector is concerned, a sufficient condition for the validity of the bound is $M_i > M_W$, which is satisfied in nature. The same applies to the quark sector, where $m_{T_i} > m_t$ is sufficient to guarantee $\Delta M_{s,d} > (\Delta M_{s,d})_{\text{SM}}$.

Our discussion of section 6 shows that in the presence of neutral particle exchanges in box diagrams with flavour violating couplings equal to the elements of the CKM matrix, our lower bound could in principle be violated. Still we have demonstrated that our bound is satisfied for

- neutral scalar particles being exchanged in box diagrams together with neutral gauge bosons,

- neutral gauge bosons belonging to SU(2) gauge groups being exchanged in box diagrams with new charged fermions. Their negative contributions to the function $S(y)$ are always overcompensated by the corresponding positive charged gauge boson contributions,

- tree level neutral gauge boson contributions like Z'.

On the other hand,

- the contribution of a U(1) gauge boson exchanged in box diagrams is strictly negative. However, if this gauge boson can also contribute at tree level, the total contribution remains positive,

- the contributions of Majorana fermions can have both signs depending on their masses relative to the masses of the scalars exchanged in the box diagrams. However a detailed analysis [17] shows that in the MFV MSSM at low $\tan \beta$ our bound is still satisfied despite the presence of gluino contributions.

It should also be emphasized that in a more general formulation of MFV in which new operators in addition to $\mathcal{Q}_q$ are admitted to contribute to $\mathcal{H}_{\text{eff}}(\Delta B = 2)$ [4], our bound can be violated, as best illustrated by the MSSM with MFV and large $\tan \beta$ [18].

Similarly, in the case of $\Delta F = 1$ processes no analogous lower bounds on the short distance functions X, Y, Z can be derived, so that for instance the sign of X can in principle be reversed within a CMFV scenario, in agreement with the analysis of [28]. A well-known example is given by the MSSM with MFV and low $\tan \beta$, where X can be both smaller and larger than X_{SM} [11, 12].

It will be of interest to watch the future improved calculations of $F_{B_q}\sqrt{\hat{B}_{B_q}}$ with the hope that one day we will know whether the lower bound derived here is satisfied in nature. Its violation would imply the relevance of new operators in $\Delta B = 2$ transitions and/or the presence of new complex phases beyond the CKM one. A simple example for the latter case is offered by the Littlest Higgs model with T-parity where indeed $\Delta M_{s,d}$ can be smaller than $(\Delta M_{s,d})_{\text{SM}}$ [15]. Our discussion of section 6 shows that neutral particle exchanges in box diagrams could also be responsible for the violation of our bound, provided they are not overcompensated by the charged particle contributions in a given model.

Acknowledgments

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